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Global Study of Electron-Quark Unparticle Interactions

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Abstract

We perform a global fit on parity-conserving electron-quark interactions via spin-1 unparticle exchange. Besides the peculiar features of unparticle exchange due to non-integral values for the scaling dimension $d_{\mathcal{U}}$ and a non-trivial phase factor $\exp(-i d_{\mathcal{U}} \pi)$ associated with a time-like unparticle propagator, the energy dependence of the unparticle contributions in the scattering amplitudes are also taken into account. The high energy data sets taken into consideration in our analysis are from (1) deep inelastic scattering at high Q^2 from ZEUS and H1, (2) Drell-Yan production at Run II of CDF and DØ, and (3) $e^+e^- \to$ hadrons at LEPII. The hadronic data at LEPII by itself indicated a $3-4$ sigma preference of new physics over the Standard Model. However, when all data sets are combined, no preference for unparticle effects can be given. We thus deduce an improved 95% confidence level limit on the unparticle energy scale $\Lambda_{\mathcal{U}}$.

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I. INTRODUCTION

Scale invariance is a very powerful symmetry in both physics and mathematics even though it is not an exact symmetry in the quantum world. As one considers the renormalizable theories of interacting elementary particles, the classical scale invariance is broken either explicitly by some dimensional mass parameters in the theory or implicitly by renormalization effects. For example, the scale invariance is broken in the Lagrangian of the Standard Model (SM) at tree level just by a negative mass squared term in the Higgs potential. Also in massless quantum chromodynamics (QCD), renormalization effects can give rise to scaling violations phenomena through the effects of running couplings and masses. Even though one has only an approximate scale invariance at low energy physics that is described well by the SM and QCD, this cannot prevent one from imagining there might be an exact scale invariant sector at a higher energy scale that has not yet been probed by experiments.

Georgi [1], motivated by the Banks-Zaks theory [2], suggested that a scale invariant hidden sector with a nontrivial infrared fixed point could exist at high enough energy. Such a scale invariant hidden sector may couple strongly or weakly within itself, but its interactions with the SM fields are presumably weak such that an effective field theory approach can be employed. Though we are ignorant of this hidden sector above this high energy scale, we still can use the approach of effective field theory to probe its low energy effects at the TeV scale.

In Georgi's scheme [1], the scale invariant sector, or denoted as Banks-Zaks ($\mathcal{BZ}$) sector, can interact with the SM fields through a messenger sector at a high mass scale $M_{\mathcal{U}}$. Below this high mass scale $M_{\mathcal{U}}$, the non-renormalizable operators are suppressed by inverse power of $M_{\mathcal{U}}$ and schematically represented in the following form [1]

$$\frac{1}{M_{\mathcal{U}}^{d_{\rm SM}+d_{\mathcal{BZ}}-4}} \mathcal{O}_{\rm SM}\, \mathcal{O}_{\mathcal{BZ}} \; , \tag{1}$$

where $\mathcal{O}_{SM}$ and $\mathcal{O}_{\mathcal{BZ}}$ represent the SM and $\mathcal{BZ}$ fields with scaling dimensions d_{SM} and $d_{\mathcal{BZ}}$ respectively. As one scales down the theory from the higher scale $M_{\mathcal{U}}$, this hidden sector may flow to an infrared fixed point at the scale $\Lambda_{\mathcal{U}}$. Georgi coined this hidden stuff as 'unparticle' $\mathcal{U}$. Below $\Lambda_{\mathcal{U}}$ one needs to replace the operators in (1) with a new set of operators having

the form

$$C_{\mathcal{O}_\mathcal{U}} \frac{1}{M_\mathcal{U}^{d_{\rm SM}+d_{\mathcal{BZ}}-4} \Lambda_\mathcal{U}^{d_\mathcal{U}-d_{\mathcal{BZ}}}} \mathcal{O}_{\rm SM}\, \mathcal{O}_\mathcal{U} \;, \tag{2}$$

where $\mathcal{O}_\mathcal{U}$ is the unparticle operator with the scaling dimension $d_\mathcal{U}$ and $C_{\mathcal{O}_\mathcal{U}}$ is the coefficient function. For an interacting scale invariant theory, the scaling dimension $d_\mathcal{U}$, unlike the case for a canonical boson or fermion field, is not necessarily an integer or half-integer. Besides, the unparticle operator $\mathcal{O}_\mathcal{U}$ with a general non-integral scaling dimension $d_\mathcal{U}$ has a mass spectrum looked like a $d_\mathcal{U}$ number of invisible massless particles [1]. Explicit list of SM invariant operators of the form Eq.(2) was written down in Ref. [3].

There have been many phenomenological studies relevant to unparticle physics in the past several years. A recent summary on the phenomenology and a more complete list of references can be found in Ref. [4]. However, there has not been a precise study of global constraint on unparticle interactions, except for an approximated estimate of $\Lambda_\mathcal{U}$ [5, 6], based on a global study of 4-fermions contact interactions [7]. Such a naive estimate could not take into account the energy dependence of the effective 4-fermions interactions due to virtual unparticle exchange in the event-by-event basis. In this work, we redo the analysis from scratch, in which full energy dependence in the event-by-event basis is taken into account in each experimental data set. In other words, our results are more accurate and valid. We analyze the parity-conserving $\ell\ell qq$ spin-1 unparticle interactions against the high energy data sets on neutral current $\ell\ell qq$ interactions, including deep inelastic scattering at high Q^2 from ZEUS and H1, Drell-Yan production at Run II of CDF and DØ, and $e^+e^- \to$ hadrons at LEPII. In the analysis, we found that the hadronic data at LEPII showed a $3-4$ sigma preference of new physics over the SM. However, when all data sets are combined, the data showed no preference. Thus, we obtain 95 % C.L. limits on the unparticle energy scale $\Lambda_\mathcal{U}$.

One can also consider spin-0 unparticle exchanges via operators such as

$$\frac{\lambda m_f}{\Lambda_\mathcal{U}^{d_\mathcal{U}}} O_\mathcal{U} \bar f f, \qquad \frac{\lambda}{\Lambda_\mathcal{U}^{d_\mathcal{U}}} (\partial^\mu O_\mathcal{U}) \bar f \gamma_\mu f \;.$$

The former scalar unparticle exchange is expected to be proportional to the Yukawa coupling of the fermion. In the high energy data sets that we study such exchanges are negligible. On the other hand, the latter scalar unparticle exchange is non-negligible because of the derivative coupling. The limits on such a scalar unparticle exchange were obtained in Ref. [6]. The limits on scalar unparticle from Bander *et al.* [6] range from 0.46 TeV for $d_\mathcal{U} = 1.9$ to 2.1 TeV for $d_\mathcal{U} = 1.1$. Nevertheless, the limits are in general an order of magnitude smaller

than those obtained for vector unparticle exchange for the same $d_{\mathcal{U}}$. Limits for spin-1 vector exchanges are also in general more stringent than those of spin-2 exchanges.

The organization of the paper is as follows. In Sec. II we present the formulation of unparticle physics and the parameterization of the electron-quark unparticle interactions that are needed in our study. In Sec. III we describe the data sets that are used in this global analysis, including the deep inelastic scattering at high Q^2 from ZEUS and H1, the Drell-Yan production at Run II of CDF and DØ as well as the total hadronic cross section $\sigma_{\rm had}$ at LEPII. We present our analysis and results in Sec. IV and conclude in Sec. V.

II. FORMULATION OF UNPARTICLE INTERACTIONS

One way to probe the existence of unparticle physics is via the interference effects between the pure SM amplitudes and the similar ones with unparticle exchange [8, 9]. It is interesting to note that the unparticle propagator has a peculiar phase factor $\exp(-i d_{\mathcal{U}} \pi)$ associated with the time-like momentum transfer [8, 9]. This complex phase in the unparticle propagator will give rise to non-trivial interference with the propagators of SM particles without this peculiar phase factor. In this study, we investigate $\ell\ell qq$ interactions by exchange with SM gauge bosons and vector unparticles.

The effective interactions of the vector unparticle operator with SM fermions is parameterized by

$$\mathcal{L}_{\rm eff} \ni \lambda_1 \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \, \bar{f}\gamma_\mu f \, O_{\mathcal{U}}^{\mu} \; , \tag{3}$$

where f denotes a SM fermion field and λ_1 is the dimensionless effective coupling. The propagator of vector unparticle is [5, 8, 9]

$$\left[\Delta_F(P^2)\right]_{\mu\nu} = Z_{d_{\mathcal{U}}} \, (-P^2)^{d_{\mathcal{U}}-2} \, \pi_{\mu\nu}(P) \; , \tag{4}$$

with

$$Z_{d_{\mathcal{U}}} \; = \; \frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)} \qquad , \tag{5}$$

$$\pi^{\mu\nu}(P) \; = \; -g^{\mu\nu} + \frac{P^\mu P^\nu}{P^2} \qquad , \tag{6}$$

and $A_{d_{\mathcal{U}}}$ is given by

$$A_{d_{\mathcal{U}}} = \frac{16\pi^{\frac{5}{2}}}{(2\pi)^{2d_{\mathcal{U}}}} \frac{\Gamma(d_{\mathcal{U}}+\frac{1}{2})}{\Gamma(d_{\mathcal{U}}-1)\Gamma(2\, d_{\mathcal{U}})} \; .$$

We need the following different treatment about the factor of $(-P^2)^{d_{\mathcal{U}}-2}$ in the propagator

$$(-P^2)^{d_{\mathcal{U}}-2} = \begin{cases} |P^2|^{d_{\mathcal{U}}-2} & \text{if } P^2 \text{ is negative and real,} \\ |P^2|^{d_{\mathcal{U}}-2} e^{-i d_{\mathcal{U}} \pi} & \text{for positive } P^2 \text{ with an infinitesimal } i0^+. \end{cases} \tag{7}$$

Therefore, the s-channel propagator has the nontrivial phase factor but the t- and u- channel propagators do not. We have imposed the requirement that the spin-1 unparticle propagator is transverse, as indicated by the tensor structure of the projection operator in Eq. (6). If the more restricted conformal invariance is assumed for the unparticle sector, the following slightly more complicated tensor structure for the projection operator must be used [10]:

$$\pi^{\mu\nu}(P) = -g^{\mu\nu} + \frac{2\,(d_{\mathcal{U}}-2)}{(d_{\mathcal{U}}-1)} \frac{P^\mu P^\nu}{P^2} \quad . \tag{8}$$

However, in the case of massless external fermions, both forms of the projection operators lead to the same effective $\ell\ell qq$ operators. A similar remark can be made to the spin-2 unparticle propagator as well. The second term in (8) does give rise to nontrivial effects in the case where the external fermion mass cannot be ignored, for example in the $B_s\overline{B}_s$ system [11]. As discussed by Georgi in his original works [1, 8], the key feature of unparticle is scale invariance. However, no one has been able to find a physically sensible interacting theory which has scale invariance but not conformal. [1] Therefore, one generally expects there is a close relation between scale invariance and being conformal. If the scale invariance is extended to be conformal, the value of $d_{\mathcal{U}}$ imposed by unitarity has to be larger than 3 for a vector unparticle propagator [13]. Hence, for completeness, in our numerical analysis presented in Sec. IV, we will consider the range of $d_{\mathcal{U}}$ varying from 1 to 4.

When the scaling dimension $d_{\mathcal{U}}$ is larger than 2, the propagator factor was shown to be modified [14]. The form of the propagator depends on the UV completion, because counter terms have to be included such that the dependence on the UV scale can become mild. In [14], it was shown that the dependence on UV scale is only logarithmic when $d_{\mathcal{U}} \to 2$. We anticipate that when appropriate counter terms are added the dependence on the UV scale could be more severe than being logarithmic for other values of $d_{\mathcal{U}} > 2$, namely that the divergence is linear for $d_{\mathcal{U}} = 3$ and more than linear for $d_{\mathcal{U}} > 3$.

[1] We note that a non-conformal but scale-invariant two-dimensional elasticity theory was constructed in Ref. [12]. However the theory is non-unitary.

To be specific, consider the parton subprocess of $q\bar{q} \to e^+e^-$. After including the virtual exchange of spin-1 unparticle, the amplitude squared (without averaging initial spins or colors) is given by

$$\sum |\mathcal{M}|^2 = 4\hat{u}^2 \left(|M^{eq}_{LL}(\hat{s})|^2 + |M^{eq}_{RR}(\hat{s})|^2 \right) + 4\hat{t}^2 \left(|M^{eq}_{LR}(\hat{s})|^2 + |M^{eq}_{RL}(\hat{s})|^2 \right) , \tag{9}$$

with

$$M^{eq}_{\alpha\beta}(\hat{s}) = \lambda_1^2 Z_{d_\mathcal{U}} \frac{1}{\Lambda_\mathcal{U}^2} \left(-\frac{\hat{s}}{\Lambda_\mathcal{U}^2} \right)^{d_\mathcal{U}-2} + \frac{e^2 Q_e Q_q}{\hat{s}} + \frac{e^2 g^e_\alpha g^q_\beta}{\sin^2\theta_\mathrm{w} \cos^2\theta_\mathrm{w}} \frac{1}{\hat{s} - M_Z^2 + i M_Z \Gamma_Z} , \tag{10}$$

where the $M^{eq}_{\alpha\beta}(\hat{s})$ is the tree level reduced amplitude and the subscripts stand for the chiralities of electrons(α) and quarks(β). In the reduced amplitude, $\hat{s}$ is the subprocess center-of-mass energy squared, $g^f_L = T_{3f} - Q_f \sin^2\theta_\mathrm{w}$ and $g^f_R = -Q_f \sin^2\theta_\mathrm{w}$ with T_{3f} and Q_f being the third component of the $SU(2)_L$ and the electric charge of the fermion f in units of the proton charge respectively, $\sin\theta_\mathrm{w}$ is the Weinberg angle and $e^2 = 4\pi\alpha_\mathrm{em}$.

To facilitate our analysis in the next section, we introduce a parameter ϵ by letting

$$\epsilon \equiv \left(\frac{\lambda_1}{\Lambda_\mathcal{U}^{d_\mathcal{U}-1}} \right)^2 , \tag{11}$$

in order to get a term similar to four-fermion contact interactions. By setting $\epsilon = 0$ we recover the original SM amplitude. For simplicity, we assume the unparticle sector is flavor- and chirality-blind. For the s-channel amplitude $\hat{s} > 0$, we have to insert the phase factor $\exp(-i d_\mathcal{U} \pi)$ into the unparticle four-fermion contact term and it will interfere with the other terms from the photon and Z boson exchange in a peculiar way for non-integral $d_\mathcal{U}$. On the other hand, when we consider non-s-channel processes, like $eq \to eq$ at HERA, we must replace the Mandelstam variable $\hat{s}$ with $\hat{t}$ or $\hat{u}$ and drop the Breit-Wigner width $iM_Z\Gamma_Z$ in the Z boson propagator. Equation (10) augments the SM amplitudes with the unparticle physics contribution in the form of an extra four-fermion contact term. We also note that for non-integral $d_\mathcal{U}$ the contact term in Eq.(10) has a nontrivial energy dependence through the factor $\left(-\frac{\hat{s}}{\Lambda_\mathcal{U}^2} \right)^{d_\mathcal{U}-2}$. Such a contact term is widely used in the model construction associated with new physics. Through a global fit of data sets we can judge if new physics can be discerned in current experiments through the contact term [7].

III. EXPERIMENTAL DATA SETS

A. HERA data

We adopt the neutral current deep inelastic scattering (DIS) e^-p data from ZEUS and e^+p scattering data from H1 [15], both measured at a center-of-mass energy $\sqrt{s} \approx 318$ GeV with an integrated luminosity of 169 pb^{-1} and 65.2 pb^{-1}, respectively.

The commonly used kinematic variables in DIS are x, y, and Q^2, which are related by

$$x = \frac{Q^2}{2p\cdot(k-k')}\,, \qquad y = \frac{Q^2}{sx}\,. \tag{12}$$

Here k and k' are the four-momentum of the incoming and outgoing leptons, p is the four-momentum of the proton and Q^2 is minus the square of the momentum transfer

$$Q^2 = -(k-k')^2 = sxy\,. \tag{13}$$

On analyzing ZEUS data, we use the reduced cross section $\tilde{\sigma}$ defined as

$$\tilde{\sigma}(e^-p) = \frac{xQ^4}{2\pi\alpha_{\rm em}^2}\frac{1}{1+(1-y^2)}\frac{d^2\sigma(e^-p)}{dxdQ^2}\,, \tag{14}$$

with the double differential cross section

$$\begin{aligned}\frac{d^2\sigma(e^-p)}{dxdQ^2} &= \frac{1}{16\pi}\left\{\sum_q f_q(x)\left[|M_{LL}^{eq}(\hat{t})|^2+|M_{RR}^{eq}(\hat{t})|^2+(1-y)^2(|M_{LR}^{eq}(\hat{t})|^2+|M_{RL}^{eq}(\hat{t})|^2)\right]\right.\\ &+\left.\sum_{\bar{q}} f_{\bar{q}}(x)\left[|M_{LR}^{eq}(\hat{t})|^2+|M_{RL}^{eq}(\hat{t})|^2+(1-y)^2(|M_{LL}^{eq}(\hat{t})|^2+|M_{RR}^{eq}(\hat{t})|^2)\right]\right\}, \end{aligned}\tag{15}$$

where $f_{q/\bar{q}}(x)$ are parton distribution functions. We use CTEQ (v.6) parton distribution functions wherever they are needed. The reduced amplitudes $M_{\alpha\beta}^{eq}$ are given by Eq. (10). On the other hand, on analyzing the H1 data we use the single differential cross section $d\sigma(e^+p)/dQ^2$ by interchanging $(LL \leftrightarrow LR, RR \leftrightarrow RL)$ in the reduced amplitudes $M_{\alpha\beta}^{eq}$ in Eq. (15) and then integrating over the x variable.

We first calculate the reduced cross section $\tilde{\sigma}$ of the SM contributions and normalize it to the whole data sets to determine the overall scale factor C, which is pretty close to 1. We then include the unparticle contributions into $\tilde{\sigma}$ and multiply it by the scale factor C determined from the previous step. For ZEUS data, the reduced cross section used in the minimization procedure is given by

$$\tilde{\sigma}^{\rm th} = C\left(\tilde{\sigma}^{\rm SM}+\tilde{\sigma}^{\rm interf}+\tilde{\sigma}^{\rm unpart}\right)\,, \tag{16}$$

where $\tilde{\sigma}^{\rm interf}$ is the interference cross section between the SM and the unparticle four-fermion interactions and $\tilde{\sigma}^{\rm unpart}$ is the cross section due to the unparticle interactions alone. We then compare the corrected theoretical values $\tilde{\sigma}^{\rm th}$ with the experimental results. Similarly, when treating the H1 data, we follow the same minimization procedure for the reduced cross section with the single differential one.

B. Tevatron: Drell-Yan Process

We use the preliminary Run II data of Drell-Yan (DY) production extracted from the CDF and DØ figures [16]. Both are measured in the form of $d\sigma/dM_{ee}$, where M_{ee} is the invariant mass of the electron-positron pair. The double differential cross section which includes the contributions of the spin-1 unparticle interactions is given by

$$\frac{d^2\sigma}{dM_{ee}dy} = K\frac{M_{ee}^3}{72\pi s}\sum_q f_q(x_1)f_{\bar{q}}(x_2)\left(|M_{LL}^{eq}(\hat{s})|^2 + |M_{LR}^{eq}(\hat{s})|^2 + |M_{RL}^{eq}(\hat{s})|^2 + |M_{RR}^{eq}(\hat{s})|^2\right)\ , \tag{17}$$

where the amplitudes $M_{\alpha\beta}^{eq}$ are given by Eq. (10). In Eq. (17), $\sqrt{s} = 1.96$ TeV is the center-of-mass energy of the $p\bar{p}$ collisions, $\hat{s} = M_{ee}^2$, y is the rapidity of the electron-positron pair and $x_{1,2} = M_{ee}\, e^{\pm y}/\sqrt{s}$. We will numerically integrate over the rapidity y distribution in our analysis. The QCD K-factor of Drell-Yan production is known to 1-loop as

$$K = 1 + \frac{\alpha_s(M_{ee}^2)}{2\pi}\frac{4}{3}\left(1 + \frac{4}{3}\pi^2\right)\ . \tag{18}$$

With this K factor, the overall cross section normalization agrees with the Tevatron data in the vicinity of the Z-peak.

C. LEP data

The LEP Electroweak Working Group (LEPEW) combined the data of total hadronic cross section from the four LEP collaborations at energies from 130 GeV to 209 GeV [17]. In the LEPEW report, they noted that the ratio of the measured cross sections to the SM expectations, averaged over all available energies, showed an approximate 1.7σ excess. We also see this effect in our fits.

In the report, both the experimental cross sections and the SM predictions are given. Since the predictions given in the report do not take into account unparticle interactions,

we do the calculation by first normalizing our tree-level SM results to the predictions given in the report and then multiplying this scaling ratio to the new cross sections that include the SM and the unparticle interactions.

At leading order in the electroweak interactions, the total hadronic cross section for $e^+e^- \to q\bar{q}$, summed over all flavors $q = u, d, s, c, b$, is given by

$$\sigma_{\rm had}(s) = K \sum_q \frac{s}{16\pi} \left[|M^{eq}_{LL}(s)|^2 + |M^{eq}_{LR}(s)|^2 + |M^{eq}_{RL}(s)|^2 + |M^{eq}_{RR}(s)|^2 \right] , \tag{19}$$

where $M^{eq}_{\alpha\beta}$ is given by Eq. (10). The prefactor K is the QCD correction known to 3-loop as

$$K = 1 + \left(\frac{\alpha_s}{\pi}\right) + 1.409 \left(\frac{\alpha_s}{\pi}\right)^2 - 12.77 \left(\frac{\alpha_s}{\pi}\right)^3 . \tag{20}$$

IV. ANALYSIS AND RESULTS

Because of severe experimental constraints on intergenerational transitions like $K \to \mu e$ we restrict our discussions to first generation contact terms. Only where required by particular data (e.g. the muon sample of Drell-Yan production at the Tevatron) shall we assume universality of contact terms between e and μ.

The effect of unparticle in the scattering amplitude for $q\bar{q} \to e^+e^-$ is explicitly given in Eq. (10) and similar formulas for other cases. In order to linearize the fitting procedures we use $\epsilon = \lambda_1^2/\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}}-2}$ of Eq. (11). The deviation from the SM is parameterized by $\epsilon Z_{d_{\mathcal{U}}}(-\hat{s})^{d_{\mathcal{U}}-2}$. The predictions by the model are then compared with the experimental data from Tevatron, LEP and HERA, as described above. We then calculate the χ^2 as a function of the parameter ϵ. We use MINUIT to minimize the χ^2 function with respect to ϵ, so as to obtain the minimum $\chi^2_{\rm min}$, which occurs at a particular $\epsilon_{\rm min}$. At the same time, we can calculate the 95% range of ϵ given by the following

$$0.95 = \frac{\int_0^{\epsilon_{95}} \exp\left[-\Delta\chi^2(\epsilon)\right]\, d\epsilon}{\int_0^{\infty} \exp\left[-\Delta\chi^2(\epsilon)\right]\, d\epsilon} , \tag{21}$$

where $\Delta\chi^2(\epsilon) \equiv \chi^2(\epsilon) - \chi^2_{\rm min}$. Here we use the fact that ϵ only takes on positive physical values.

In a previous publication [5], we gave the approximate limits on $\Lambda_{\mathcal{U}}$ based on an analysis on 4-fermion contact interactions [7]. Those estimates could not take into account the energy dependence of the unparticle contribution, as shown in Eq. (10). This is the most important

TABLE I: Fitted values of $\epsilon \equiv \lambda_1^2/\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}}-2}$ of each experimental set and the combined set. The 95% C.L. lower limit on $\Lambda_{\mathcal{U}}$ for each $d_{\mathcal{U}}$ is obtained by choosing $\lambda_1 = 1$ in the value of ϵ_{95} defined by Eq. (21).

$d_{\mathcal{U}}$	Fitted parameter $\epsilon \equiv \frac{\lambda_1^2}{\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}}-2}}$				$\Lambda_{\mathcal{U}}$ $(\lambda_1 = 1)$
	Tevatron DY	HERA DIS	LEP $q\bar{q}$	Combined	(TeV)
1.1	$\left(0.79^{+0.61}_{-0.58}\right) \times 10^{-3}$	$\left(-0.035^{+0.16}_{-0.15}\right) \times 10^{-3}$	$\left(-2.65^{+0.74}_{-0.65}\right) \times 10^{-3}$	$(-0.023 \pm 0.15) \times 10^{-3}$	5.5×10^{14}
1.3	$\left(0.52^{+0.39}_{-0.34}\right) \times 10^{-3}$	$\left(-0.24^{+1.00}_{-0.98}\right) \times 10^{-4}$	$\left(-0.83^{+0.21}_{-0.18}\right) \times 10^{-3}$	$\left(-0.18^{+0.98}_{-0.96}\right) \times 10^{-4}$	1.7×10^3
1.5	$(0.00 \pm 0.17) \times 10^{-3}$	$\left(-0.11^{+0.57}_{-0.56}\right) \times 10^{-4}$	$\left(2.81^{+0.46}_{-0.55}\right) \times 10^{-4}$	$\left(-0.23^{+0.84}_{-0.76}\right) \times 10^{-4}$	7.3
1.7	$\left(-0.31^{+0.20}_{-0.30}\right) \times 10^{-4}$	$(-0.015 \pm 0.25) \times 10^{-4}$	$\left(0.40^{+0.089}_{-0.10}\right) \times 10^{-4}$	$\left(-0.024^{+0.16}_{-0.72}\right) \times 10^{-4}$	2.1
1.9	$\left(-0.15^{+0.11}_{-0.12}\right) \times 10^{-5}$	$\left(0.077^{+0.55}_{-0.52}\right) \times 10^{-5}$	$\left(0.36^{+0.092}_{-0.11}\right) \times 10^{-5}$	$\left(-0.003^{+0.091}_{-0.11}\right) \times 10^{-5}$	1.7
2.1	$\left(0.38^{+0.34}_{-0.30}\right) \times 10^{-6}$	$\left(-0.10^{+0.28}_{-0.30}\right) \times 10^{-5}$	$\left(-1.16^{+0.34}_{-0.30}\right) \times 10^{-6}$	$\left(0.01^{+0.29}_{-0.26}\right) \times 10^{-6}$	0.62
2.5	$\left(0.16^{+0.15}_{-0.47}\right) \times 10^{-6}$	$\left(-0.13^{+0.23}_{-0.28}\right) \times 10^{-5}$	$\left(0.82^{+0.14}_{-0.18}\right) \times 10^{-6}$	$\left(-0.27^{+0.20}_{-0.11}\right) \times 10^{-6}$	0.14
2.9	$\left(-0.22^{+0.33}_{-1.06}\right) \times 10^{-8}$	$\left(-0.10^{+0.18}_{-0.25}\right) \times 10^{-6}$	$\left(2.39^{+0.64}_{-0.74}\right) \times 10^{-8}$	$\left(-0.05^{+2.40}_{-3.39}\right) \times 10^{-9}$	0.16
3.1	$\left(0.28^{+0.09}_{-0.31}\right) \times 10^{-8}$	$\left(0.46^{+1.45}_{-0.91}\right) \times 10^{-7}$	$\left(-1.03^{+0.32}_{-0.28}\right) \times 10^{-8}$	$\left(-0.06^{+1.50}_{-0.79}\right) \times 10^{-9}$	0.10
3.5	$\left(0.58^{+0.18}_{-0.28}\right) \times 10^{-9}$	$\left(0.34^{+1.77}_{-0.80}\right) \times 10^{-7}$	$\left(-1.15^{+0.28}_{-0.22}\right) \times 10^{-8}$	$\left(0.58^{+0.18}_{-0.27}\right) \times 10^{-9}$	0.066
3.9	$\left(0.85^{+0.58}_{-3.30}\right) \times 10^{-11}$	$\left(0.24^{+2.27}_{-0.72}\right) \times 10^{-8}$	$\left(0.50^{+0.14}_{-0.16}\right) \times 10^{-9}$	$\left(0.89^{+0.56}_{-3.22}\right) \times 10^{-11}$	0.072

improvement of this work that we have taken into account the energy dependence of each experimental set as well as in each event (such as in Drell-Yan production the $\hat{s}$ of each event is different.)

We show in Tables I and II our main results. The difference between Table I and II is that the fittings in Table II are without $q\bar{q}$ pair production at LEP2. This is because we found that when we fitted the unparticle ϵ term to the LEP2 $q\bar{q}$ data alone, the fitted values showed a $3-4\ \sigma$ deviation from zero. Indeed, it was reported in Ref. [17] that the hadronic cross sections from $\sqrt{s} = 130 - 207$ GeV are systematically higher (on the average 1.7σ) than the SM predictions. Therefore, by combining all the energy data from LEP2 we obtain the fits $3-4\ \sigma$ deviation from the SM. Nevertheless, the combined Tevatron DY, HERA DIS and LEP $q\bar{q}$ results do not show any appreciable deviation from the SM, as the LEP2 data are never dominant. The lower limits on $\Lambda_{\mathcal{U}}$ do not change significantly between Tables I and II. We also noticed that at small $d_{\mathcal{U}} \approx 1.1 - 1.5$ the limit is dominated by the HERA

TABLE II: Same as Table I but without the hadronic data set of LEP.

$d_{\mathcal{U}}$	Fitted parameter $\epsilon \equiv \frac{\lambda_1^2}{\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}}-2}}$ Tevatron DY + HERA DIS	$\Lambda_{\mathcal{U}}$ ($\lambda_1 = 1$) (TeV)
1.1	$(0.023 \pm 0.15) \times 10^{-3}$	3.3×10^{14}
1.3	$\left(0.26^{+0.95}_{-0.93}\right) \times 10^{-4}$	1.4×10^{3}
1.5	$(-0.10 \pm 0.54) \times 10^{-4}$	9.9
1.7	$\left(-0.19^{+0.14}_{-0.16}\right) \times 10^{-4}$	2.8
1.9	$\left(-0.14^{+0.10}_{-0.11}\right) \times 10^{-5}$	2.0
2.1	$\left(0.36^{+0.33}_{-0.30}\right) \times 10^{-6}$	0.53
2.5	$\left(-0.17^{+0.46}_{-0.15}\right) \times 10^{-6}$	0.14
2.9	$\left(-0.22^{+0.33}_{-1.07}\right) \times 10^{-8}$	0.16
3.1	$\left(0.28^{+0.09}_{-0.31}\right) \times 10^{-8}$	0.10
3.5	$\left(0.58^{+0.18}_{-0.27}\right) \times 10^{-9}$	0.066
3.9	$\left(0.85^{+0.58}_{-3.30}\right) \times 10^{-11}$	0.072

data while for larger $d_{\mathcal{U}} \approx 1.7 - 1.9$ the limit is dominated by the Tevatron data. The 95% C.L. lower limits on $\Lambda_{\mathcal{U}}$, assuming $\lambda_1 = 1$ and $d_{\mathcal{U}} = 1.1 - 1.9$, are

$$\Lambda_{\mathcal{U}} = \begin{cases} 1.7 - 5.5 \times 10^{14}\ \text{TeV} & \text{w/ LEP2 data} \\ 2.0 - 3.3 \times 10^{14}\ \text{TeV} & \text{w/o LEP2 data} \end{cases} \tag{22}$$

On the other hand, for $d_{\mathcal{U}} = 2.1 - 2.9$ the limits on $\Lambda_{\mathcal{U}}$ are

$$\Lambda_{\mathcal{U}} = \begin{cases} 0.14 - 0.62\ \text{TeV} & \text{w/ LEP2 data} \\ 0.14 - 0.53\ \text{TeV} & \text{w/o LEP2 data} \end{cases} \tag{23}$$

whereas for $d_{\mathcal{U}} = 3.1 - 3.9$ the limits on $\Lambda_{\mathcal{U}}$ are

$$\Lambda_{\mathcal{U}} = 0.066 - 0.10\ \text{TeV}\ . \tag{24}$$

If we ignore the requirement of conformality, $d_{\mathcal{U}} = 1 - 2$ gives the most severe bounds on $\Lambda_{\mathcal{U}} \sim 10^{14} - 2$ TeV, making observation at the LHC very difficult. On the other hand, for $d_{\mathcal{U}} = 2 - 4$ the bounds are at the electroweak scale ($0.1 - 1$ TeV), making potential observations at the LHC. If conformality is to be maintained for vector unparticle, the

unitarity constraint requires $d_{\mathcal{U}} > 3$, for which the bounds from the experimental data are very mild, only $0.07 - 0.1$ TeV. Following Ref. [18] a consistency condition can be imposed to maintain the conformality and unitarity

$$8\frac{\sin(-\pi d_{\mathcal{U}})\Gamma(2-d_{\mathcal{U}})}{(4\pi)^{(2d_{\mathcal{U}}-2)}\Gamma(d_{\mathcal{U}})}\frac{2}{\pi}\left(\frac{q^2}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}} < 1\ , \tag{25}$$

where $q^2 \leq \hat{s}_{\rm max}$ and $\hat{s}_{\rm max}$ depends on the experimental conditions. It was shown that [18] the bounds on $\Lambda_{\mathcal{U}}$ are better constrained by this theoretical argument than the experimental data for $d_{\mathcal{U}} \gtrsim 1.5$. One can extend the above theoretical condition to $d_{\mathcal{U}} > 2$, and we obtain, by putting $\hat{s}_{\rm max} \simeq (2\ \text{TeV})^2$ for the Tevatron,

$$\begin{aligned} \Lambda_{\mathcal{U}} &\gtrsim 1.2 - 2\ \text{TeV} \qquad \text{for} \quad d_{\mathcal{U}} = 1.1 - 1.9\ , \\ &\gtrsim 0.6 - 1\ \text{TeV} \qquad \text{for} \quad d_{\mathcal{U}} = 2.1 - 2.9\ , \\ &\gtrsim 0.3 - 0.5\ \text{TeV} \qquad \text{for} \quad d_{\mathcal{U}} = 3.1 - 3.9\ . \end{aligned}$$

We can see that bounds from experimental data are more severe for small $d_{\mathcal{U}} = 1 - 2$ while the conformality places stronger limits on $\Lambda_{\mathcal{U}}$ for $d_{\mathcal{U}} \gtrsim 2$; especially for $d_{\mathcal{U}} > 3$ the limits are an order of magnitude stronger than the experimental bounds.

V. CONCLUSIONS

We have investigated the global constraint on the parity-conserving $\ell\ell qq$ spin-1 unparticle interactions by comparing the theoretical predictions given by unparticle against high energy data sets, including the HERA high-Q^2 neutral-current data, Drell-Yan production at the Tevatron, and the LEPII hadronic cross sections. Overall, the combined data sets do not show any preference of unparticle over the SM, although the LEPII data alone does show some preference. Thus, we obtain 95% C.L. lower limits on the unparticle scale $\Lambda_{\mathcal{U}}$. It ranges widely from 2 TeV to $O(10^{14})$ TeV for $d_{\mathcal{U}} = 1.1 - 1.9$, whereas from $0.14 - 0.62$ ($0.066 - 0.1$) TeV for $d_{\mathcal{U}} = 2.1 - 2.9$ ($3.1 - 3.9$), depending sensitively on the unparticle scaling dimension $d_{\mathcal{U}}$. This work, by analyzing all the three data sets from scratch, is a real improvement over the previous naive estimate based on rescaling the results from the conventional 4-fermion contact interactions. The energy dependence of the unparticle contributions in the scattering amplitudes are taken into account appropriately. The unitarity constraint of $d_{\mathcal{U}} > 3$ for vector unparticle has pushed the limit of the unparticle scale $\Lambda_{\mathcal{U}}$ down to the

electroweak scale. Nevertheless, following Ref. [18] we can use the theoretical condition for conformality and unitarity, and place a useful constraint on $\Lambda_{\mathcal{U}}$ for $d_{\mathcal{U}} > 3$. In other words, the conformality helps improving the bounds, especially for $d_{\mathcal{U}} > 3$.

The limits obtained in this paper serve as the most precise ones for the parity-conserving spin-1 unparticle interactions. If parity-violating interactions are included, the parity-violating data sets, such as atomic parity violation, have to be taken into account. Extensions to the spin-2 unparticle interactions are straight-forward, but in general the limits so obtained are less stringent than those from the spin-1 case obtained in this work.

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